

EXERCISES

In the notes I define addition and multiplication of cardinals. Here you can explore subtraction and division of cardinals.

Exercise 1. (*Subtraction of Cardinals*) Let d and e be two given cardinal numbers. Prove that the solutions of $d + d_1 = e$ are given as follows:

- (1) for $d > e$ there is no solution,
- (2) for $d = e$ and d infinite, d_1 can be any cardinal number $\leq d$,
- (3) for $d < e$ and e infinite, $d_1 = e$ is the unique solution.

Exercise 2. (*Division of Cardinals*) Let d and e be two given cardinal numbers. Prove that the solutions of $dd_1 = e$ are given as follows:

- (1) for $d > e$ there is no solution,
- (2) for $d = e$ and d infinite, d_1 can be any cardinal number $\leq d$,
- (3) for $d < e$ and e infinite, $d_1 = e$ is the unique solution.

Exercise 3. If e is an infinite cardinal number and d is a cardinal number satisfying $2 \leq d \leq 2^e$, prove that $d^e = 2^e$.